

## Abstract

The neutrino is a weakly interacting, electrically neutral particle in the standard model of particle physics. These neutrinos are referred to as left-handed or active neutrinos and are classified into three flavors (electron, mu, and tau). Neutrino oscillation is the phenomenon that involves the oscillation of neutrinos between the three flavors.[1] This phenomenon is also applied the oscillation of left-handed neutrinos into right-handed (sterile) neutrinos. The sterile neutrino is a hypothetical particle that does not interact with the weak force and only interacts through the gravitational force. [1]This characteristic of the sterile neutrino makes it a very good dark matter candidate. Dark matter is a form of matter that does not interact with the electromagnetic force, which mean that it does not interact with light. The objective of this project has been to calculate the abundance of sterile neutrinos in the early universe and use this abundance to constrain the parameter space of the model using data from X-ray observations. [2]

## Diagonalization

To begin investigating the distribution of sterile neutrinos it is important to begin by establishing the formalism that will be utilized. Starting with the Lagrangian  $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_{SM}$  where  $\mathcal{L}_{SM}$  is the Lagrangian of the Standard Model and  $\mathcal{L}_N$  is the Lagrangian of the right handed neutrino and goes as:

$$\mathcal{L}_N = \bar{\nu}_R i \not{\partial} \nu_R - \nu_R^c T y_\nu LH + \frac{1}{2} \nu_R^c T \mathcal{M}_N \nu_R^c - H.C. \quad (1)$$

Where H and L are the Higgs and Lepton doublets respectively and  $\nu_R$  is the right-handed neutrino.  $y_\nu$  is the Yukawa coupling [1]. To begin the diagonalization of  $\mathbb{M}$  consider the terms contained in  $\mathcal{L}_N$ :

$$\frac{1}{2} (\nu_L^T \quad \nu_R^c T) \mathbb{M} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \mathbb{M} = \begin{pmatrix} 0 & m_D^T \\ -m_D & \mathcal{M}_N \end{pmatrix} \quad (2)$$

The eigenvalues of  $\mathbb{M}$  go as

$$\lambda = \frac{1}{2} (\mathcal{M}_N \pm \mathcal{M}_N \sqrt{\mathcal{M}_N^2 - 4(-m_D m_D^T)}) \quad (3)$$

and by Taylor expanding  $\lambda$  becomes

$$\lambda = \frac{1}{2} \left( \mathcal{M}_N \pm \left( \mathcal{M}_N + \frac{2m_D m_D^T}{\mathcal{M}_N} \right) \right) \quad (4)$$

## Diagonalization Cont.

and the eigenvalues will go as  $\lambda = \mathcal{M}_N$  and  $\lambda = \frac{-m_D^T m_D}{\mathcal{M}_N}$ . From the eigenvalues the diagonalized form of  $\mathbb{M}$  goes as

$$\mathbb{M}_D = \tilde{U} \mathbb{M} \tilde{U}^T \quad (5)$$

and  $\tilde{U}$  is the matrix use to diagonalize. and is defined as:

$$\tilde{U} = \begin{pmatrix} \mathbb{I} & \theta^\dagger \\ -\theta & \mathbb{I} \end{pmatrix} \quad (6)$$

where  $\theta$  is defined as the mixing angle between neutrinos and goes as:  $m_D \mathcal{M}_N^{-1}$  [1]. The diagonalized result is:

$$\begin{pmatrix} -m_D^{\text{diag}T} \mathcal{M}_N^{-1} m_D^{\text{diag}} & 0 \\ 0 & \mathcal{M}_N \end{pmatrix} \quad (7)$$

and written in a more convenient notation:

$$\mathbb{M} = \begin{pmatrix} -\mathcal{M}_\nu & 0 \\ 0 & \mathcal{M}_N \end{pmatrix} \quad (8)$$

where  $\mathcal{M}_\nu = m_D^{\text{diag}} \mathcal{M}_N m_D^{\text{diag}}$ . The matrix representation of  $\mathcal{M}_N$  goes as:

$$\begin{pmatrix} M_0 & m & \\ m & M_2 & \\ & & M_3 \end{pmatrix} \quad (9)$$

$\mathcal{M}_N^{\text{diag}}$  takes the form of  $\text{diag}(M_1, M_2, M_3)$  where  $M_1 \approx M_0 - \frac{m^2}{M_2}$  [1].

## Abundance Density

Using the resulting masses and the mixing angle from the diagonalization of  $\mathbb{M}$  to calculate the distribution of sterile neutrinos in the early universe [1]. Starting with the Boltzmann Equation:

$$\frac{dn_{\nu_{R1}}}{dt} + 3Hn_{\nu_{R1}} = C_{\nu_{R1}} \quad (10)$$

where the collision term  $C_{\nu_{R1}}$  goes as

$$C_{\nu_{R1}} = \mathcal{P}(\nu_{R2} \rightarrow \nu_{R1}) (\gamma_{\nu_{R2}}^{\text{col}} + \gamma_{\nu_{R2}}^{\text{ID}}) \quad (11)$$

Where

$$\gamma_{\nu_{R2}}^{\text{col}} = \frac{T}{64\pi^4} \int_{s_{\text{min}}}^{\infty} ds \hat{\sigma} \sqrt{s} k_1 \left( \frac{\sqrt{s}}{T} \right) \quad (12)$$

and

$$\gamma_{\nu_{R2}}^{\text{ID}} = \frac{M_2^2 T}{\pi^2} \Gamma(\nu_{R2} \rightarrow LH) k_1 \left( \frac{M_2}{T} \right) \quad (13)$$

The two collisional terms can be summed and simplified to  $\gamma_{\nu_{R2}}$ . Then additionally defining the probability  $\mathcal{P}(\nu_{R2} \rightarrow \nu_{R1})$  as  $\frac{1}{2} \sin^2(2\theta_N)$  where  $\theta_N$  is defined as the mixing angle between between  $\nu_{R2}$  and  $\nu_{R1}$  [1].

## Abundance Density Cont.

$\theta_N$  is equivalent to  $\frac{m}{M_2}$ .  $\hat{\sigma}$  is the reduced cross section and  $k_1$  is the modified Bessel function of the first kind.  $\Gamma(\nu_{R2} \rightarrow LH)$  is known as the decay width and goes as:  $(y_\nu y_\nu^\dagger)_{22} \frac{M_2}{8\pi}$ . Solving the Boltzmann equation will result in the yield which goes as:

$$Y_{N1}(T=0) = \int_0^\infty dT \mathcal{P}(\nu_{R2} \rightarrow \nu_{R1}) \frac{\gamma_{R2}}{sHT} \quad (14)$$

H is the Hubble expansion rate and s is the entropy density [2]. The result from the yield can be rescaled by defining  $\tilde{Y}_{N1}^0$  to be:

$$\tilde{Y}_{N1}^0 = \frac{Y_{N1}}{\mathcal{P}(\nu_{R2} \rightarrow \nu_{R1}) (y_\nu y_\nu^\dagger)_{22}} \quad (15)$$

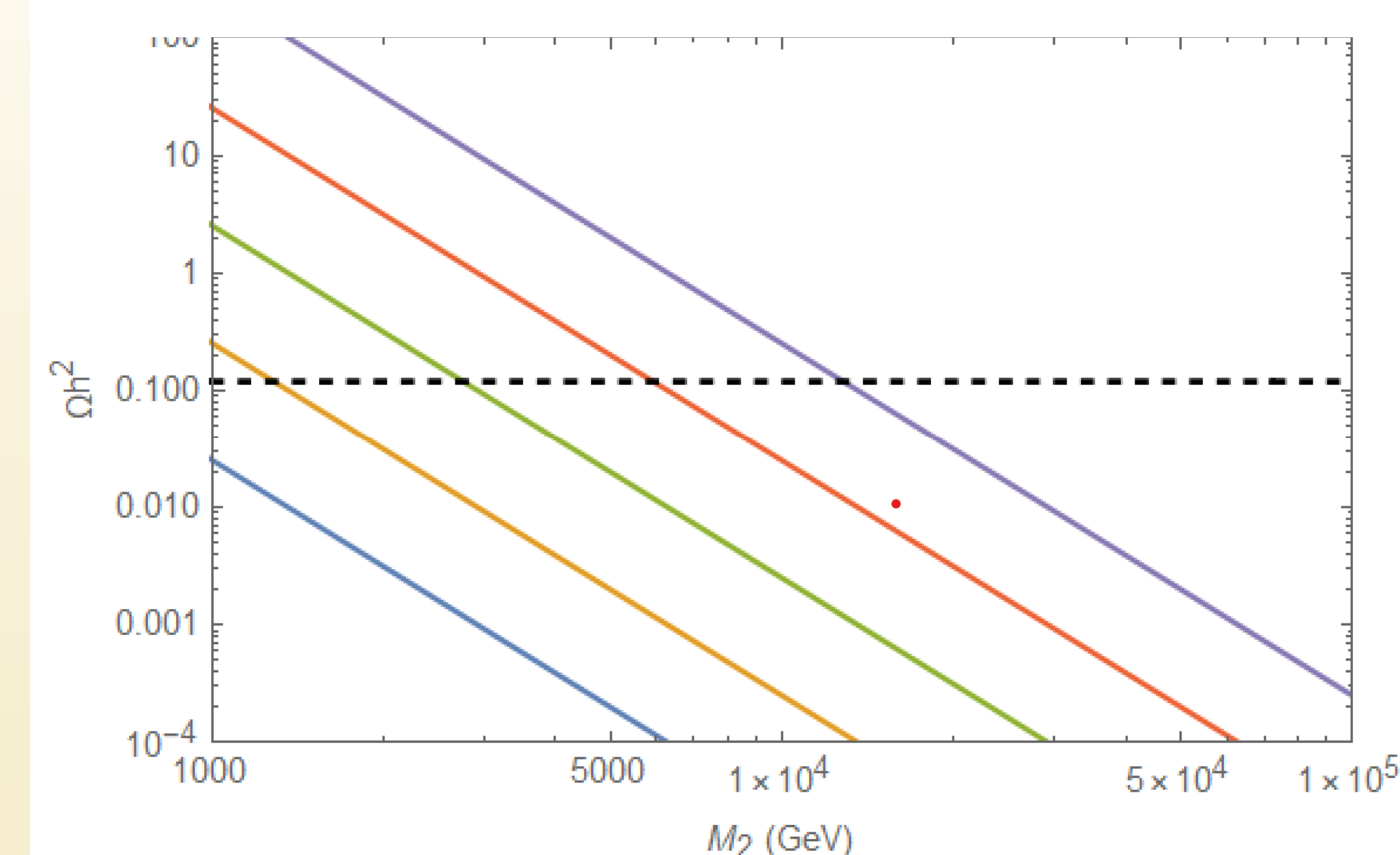


Figure 1 shows the relic abundance of sterile neutrinos vs  $M_2$  for varying values of  $M_1$  where from blue to violet the  $M_1$  values are 10 keV, 100keV, 1 MeV, 10 MeV, and 100 MeV. The dashed line represents the relic abundance of dark matter.

The rescaled parameter is then plugged into the following equation:

$$\Omega_{N1} h^2 \approx 0.12 \left[ \frac{\sin^2(2\theta_N)}{8.8 * 10^{-3}} \right] \left[ \frac{|y_\nu^{\text{diag}}|_{22}^2}{10^{-13}} \right] \left[ \frac{M_1}{\text{keV}} \right] \left[ \frac{\tilde{Y}_{N1}^0}{10^{12}} \right] \quad (16)$$

The resulting term  $\Omega_{N1} h^2$  is the relic abundance of sterile neutrinos [1].

## Conclusion

The contours of the relic abundance for the choices of  $M_1$  cross the expected relic abundance of 0.12. This makes the sterile neutrino a viable dark matter candidate. Future implications would include investigating the decay rates of sterile neutrinos into X-rays which would establish constraints on the system.

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## References

- [1] T. G. G. Richard Slansky, Stuart Raby and N. G. Cooper, Neutrinos in the standard model, (1997).
- [2] K. Kadota and K. Kaneta, Sterile neutrino dark matter from right-handed neutrino oscillations, Physical Review D 97, 10.1103/physrevd.97.115021 (2018).
- [3] E. Hubble, A relation between distance and radial velocity among extra-galactic nebulae, PNAS 15, 168 (1929).